The workbook *A Companion to Discover Linear Algebra* is a comprehensive set of notes, examples, and problems intended to be used as a supplement to the text. It covers topics such as systems of equations, matrices and inverses, determinants, vector spaces and subspaces, linear independence, basis and dimension, row/column/null spaces, eigenvalues/eigenvectors, and diagonalization. In addition, it includes examples, review material for exams, and practice exam questions. The chapter summaries below provide a more detailed description of the topics included in the workbook.

Linear Algebra Workbook Summary

Chapter 1: Systems of Equations

- Consistent vs. inconsistent systems
- Unique, infinite, and no-solution cases (geometric interpretations)
- Graphing systems of equations in two variables
- Augmented matrices and elementary row operations

Chapter 2: Row Reduction & Solutions

- Row echelon form (REF) and reduced row echelon form (RREF)
- Rank, leading/free variables, homogeneous vs. nonhomogeneous systems
- Parametric form of solutions: (# parameters) = (# variables – rank)
- Example: solving systems, finding homogeneous & particular solutions

Chapter 4: Matrix Equations

- Matrix multiplication
- Writing systems as Ax = b
- General and homogeneous solutions in vector/matrix form

Chapter 5: Inverses

- Identity, inverse, singular vs. nonsingular matrices
- Properties of determinants
- Finding inverses and using them to solve systems

Chapter 6: Elementary Matrices

- Definition and use for row operations
- Equivalence statements for invertibility:
  - Ax=0 has only trivial solution
  - RREF = Identity
  - A is product of elementary matrices
  - Ax=b consistent for all b

Chapter 8: Determinants

- Cofactor expansion
- Special matrices (triangular, diagonal)
- Properties of determinants

Chapter 11–12: Vectors

- Vectors, components, zero vector
- Norm, unit vectors, distance, dot product
- Cauchy-Schwarz inequality

Chapter 15–16: Vector Spaces & Subspaces

- Vector space axioms, zero vector, negatives, closure
- Subspace test (nonempty, closed under addition & scalar multiplication)
- Spanning sets, linear combinations
- Examples: polynomials, matrices, geometric subspaces

Midterm 1 Review problems with solutions

- Systems of equations via REF/RREF
- Matrix operations, inverses, rank, free variables
- Geometric interpretations of solutions
- Vectors, unit vectors, subspaces, spans

Chapter 17: Linear Independence

- Linearly dependent vs. independent sets
- Test: solving homogeneous equation
- Reducing spanning sets to independent sets

Chapter 18: Basis & Coordinates

- Basis, ordered basis, coordinate vectors
- Examples: finding bases and coordinate representations
- Relation to polynomial and vector spaces

Chapter 20: Column, Row & Null Space

- Column space, row space, null space, nullity
- Basis-finding procedures (using REF)
- Relationship between rank, row space, and column space

Chapter 21: Eigenvalues, Eigenvectors & Diagonalization

- Definitions: eigenvalues, eigenvectors, eigenspaces
- Characteristic polynomial & equation
- Procedures to compute eigenvalues/eigenvectors
- Similar matrices and diagonalization criteria (algebraic vs. geometric multiplicity)
- Transition matri